B.Sc. 3rd Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 32112 **Course Code : SHMTH-302/C-6**

Course Title: Group Theory-I

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbol have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) If G is an abelian group, then prove that $H = \{a \in G : a^2 = e\}$ is a subgroup of G.

(b) Give an example of an infinite group in which every element is of finite order.

(c) Find the order of the element $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 1 & 4 & 2 & 6 & 9 & 10 & 3 & 5 & 8 \end{pmatrix}$ in S_{10}.

(d) Prove that the order of *An* is $\frac{n!}{2}$.

(e) Find the cyclic subgroup of $(\mathbb{Z}_{30}, +)$ generated by 25.

(f) Prove that there cannot be any epimorphism from the group $(\mathbb{Z}_{15}, +)$ onto the group $(\mathbb{Z}_8, +)$.

(g) Prove that any finite cyclic groups of same order are isomorphic.

(h) Prove that the groups $(\mathbb{Z}, +)$ and $(\mathbb{R}, +)$ are not isomorphic.

2. Answer *any four* questions: 5×4=20

(a) Let H and K be two subgroups of a group G. Prove that HK is a subgroup G if and only if $HK = KH$. 5

(b) (i) Solve the following equation in the Klein's 4-group $K - 4$:
$b^{-1}cx^2ac^3 = bac$, where $K_4 = \{e, a, b, c\}$.

(ii) Let $(G,*)$ be a finite group containing even number of elements. Prove that there exists at least one elements a, other than the identify e in G such that $a * a = e$ holds. 2+3=5

(c) (i) Prove that a finite semigroup satisfying the cancellation laws, forms a group.

(ii) Let G be a group. Show that $o(a) = o(b^{-1}ab) = o(bab^{-1})$. 3+2=5

(d) Let $n \geq 2$ and $\sigma \in S_n$ be a cycle. Them σ is a k-cycle iff order of σ is k. 5

(e) Let f be a homomorphism from the group G onto G_1. Then show that $\frac{G}{Kerf} \cong G_1$. 5

(f) (i) Show that the set of all rotations

$T(\theta)\colon (x, y) \to (x', y')$ where $x' = x\cos\theta - y\sin\theta,\ y' = x\sin\theta + y\cos\theta$ forms a group with respect to multiplication.

(ii) Write down all the elements of the group U_{10}. 4+1=5

3. Answer *any one* question: 10×1=10

(a) (i) Define the Dihedral group D_4 of order 8. Find the centre of D_4.

(ii) Let $G = \left\{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}\colon\ a, b, c \text{ are reals and } ac \neq 0\right\}$ be a group under usual matrix multiplication. Show that $N = \left\{\begin{pmatrix} 1 & c \\ 0 & 1 \end{pmatrix} : c \text{ is real}\right\}$ is a normal subgroup of G.

(iii) Prove that every group of prime order is cyclic. (2+3)+3+2=10

(b) (i) Prove that the group $\frac{4\mathbb{Z}}{12\mathbb{Z}} \cong \mathbb{Z}_3$.

(ii) Let H be a subgroup of a group G such that $[G\colon H] = 2$. Then prove that H is a normal subgroup of G. Is the converse true? Justify your answer. 4+(3+3)=10

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B.Sc. 3rd Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 32113 **Course Code : SHMTH-303-C-7**

Course Title: Numerical Methods

Time: 1 Hour 15 Minutes **Full Marks: 25**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 1×5=5

(a) Determine the number of correct (significant) digits in the number $x = 0 \cdot 4785$ given its relative error $E_r = 0 \cdot 3 \times 10^{-2}$.

(b) Show that $\Delta \log f(x) = \log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$, where Δ is the forward difference operator.

(c) Explain why the degree of Precision of Simpson's one-third quadrature formula is 3.

(d) Write down the condition of convergence of the Newton-Raphson method for solving an equation $f(x) = 0$.

(e) In the algorithm of Runge-Kutta method of order 4, write the usual expressions for K_2 and K_3.

(f) Find y when $\frac{dy}{dx} = x + y^2$ with $y(0) = 0$ by Picard's approximation method after two iterations.

(g) Find the Lagrange's interpolation polynomial fitting the data points $f(1) = 6, f(3) = 0, f(4) = 12$ for some function $f(x)$.

(h) State the condition of convergence of Gauss-Seidal iteration method for solving numerically a system of linear algebraic equations.

2. Answer *any two* questions: 5×2=10

(a) Explain the Regula-Falsi method (method of False position) in obtaining a simple real root of an equation of the form $f(x) = 0$. Why does the method is called 'Linear interpolation method'? 4+1=5

(b) Prove that the remainder in approximating a function $f(x)$ by the interpolation polynomial $\phi(x)$ using interpolating points $x_0, x_1 \ldots, x_n$ is of the form

$$(x-x_0)(x-x_1)\ldots(x-x_n)\frac{f^{(n+1)}(\xi)}{(n+1)!}$$

where ξ lies between the smallest and the largest of the numbers $x, x_0, x_1 \ldots x_n$

(c) Find by the Euler's modified method, the value of y for $x = 0 \cdot 05$ from the differential equation $\frac{dy}{dx} = x + y$. Correct up to four places of decimals with the initial condition $y = 1$ when $x = 0$.

(d) Using Simpson's one-third quadrature formula find the value of $\int_{1\cdot 2}^{1\cdot 6}\left(x+\frac{1}{x}\right)dx$; Correct up to two significant figures taking $n = 4$. Show the calculations side by side.

3. Answer *any one* question: 10×1=10

(a) (i) With an example illustrate the 'truncation error'.

(ii) Discuss the Geometrical significance of Trapezoidal rule.

(iii) With usual symbols, establish the relation $f[x_0, x_1 \ldots, x_n] = \frac{\Delta^n y_0}{n! h^n}$

where $x_r = x_0 + rh,\ r = 1, 2, \ldots, n$. 2+3+5=10

(b) (i) Describe briefly Gauss elimination method for solving a system of Linear algebraic equations without pivoting.

(ii) Given $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$. Find $y(0 \cdot 1)$ by 4th order Runge-Kutta Method (Correct up to 4 decimal places). 5+5=10

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SH-I/Math-103/GE-1/(PR)/19

B.Sc. 3rd Semester (Honours) Practical Examination, 2019-20

MATHEMATICS

Course ID : 32123 **Course Code : SHMTH-303/C-7**

Course Title: Numerical Models Lab

Instructions to the Candidates.

Candidates are required to answer the question as indicated in the card drawn by them. They are also required to write the suitable program in C programming language to solve the problem.

Marks Distribution :

Given Problem	:	8 marks
Sessional	:	4 marks
Viva voce	:	3 marks
Total	:	15 marks

1. Using Newton-Raphson's method find one real root, correct up to five places of decimals, of the equation given in the card drawn by you.

2. Using Lagrange's interpolation method find the values of y correct up to 4 places of decimals for given x. The tabular values of x and y and the value of x for which y is to be calculated are all provided in the card drawn by you.

3. Using the Trapezoidal rule, find the value of the integral, provided in the card drawn by you, taking 50 equal subintervals.

4. Using the Simpson's 1/3-rule, find the value of the integral, provided in the card drawn by you, taking 50 equal subintervals.

5. Using the Fourth-orders Runge-Kutta method find the values of y for different values of x from the given differential equation. The differential equation with initial data and the different values of x for which the y-values are to be calculated are given in the card drawn by you.

6. Using the Power method find the largest Eigen value and the corresponding Eigen vector correct up to five places of decimals from the matrix given in the card drawn by you

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B.Sc. 3rd Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 32114 **Course Code : SHMTH-304/GE-3**

Course Title: Algebra

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) If $Z+\frac{1}{Z}=\sqrt{3}$, then show that $Z^6=-1$.

(b) Show that $n^n \geq n!$ for all $n \in N$.

(c) Give an example of transitive relation which is

(i) symmetric but not reflexive and

(ii) reflexive but not symmetric.

(d) Transform the equation to remove the square from $x^3-15x^2-33x+84=0$.

(e) Apply Descartes rule of signs to examine the nature of the roots of the equation $x^4+2x^2+3x-1=0$.

(f) Use division algorithm, find integers u and v satisfying $63u+55v=1$

(g) Find the dimension of the subspace W of $\mathbb{R}^4$, where

$W=\{(x,y,z,w)\in\mathbb{R}^4 : x+2y-z=0,\ 2x+y+w=0\}$

(h) Define $T:\mathbb{R}^2\to\mathbb{R}^2$ by $T(x,y)=(2x+y,y)$, $(x,y)\in\mathbb{R}^2$ Examine whether T is linear or not? Justify your answer.

2. Answer *any four* questions: 5×4=20

(a) (i) Show that the product of all values of $\left(1+i\sqrt{3}\right)^{\frac{3}{4}}$ is 8.

(ii) State Cauchy-Schwarz inequality. 3+2=5

(b) Define linear transformation. Find out the matrix corresponding to the linear transformation $T:\mathbb{R}^3\to\mathbb{R}^2$ $s.t.$

$$T(x,y,z)=\left(2x+2y+z,\frac{-x+y+3z}{2}\right)$$

with the respect to the ordered basis $\{(0,1,1)(1,0,1)(1,1,0)\}$ of $\mathbb{R}^3$ and $\{(1,0),(1,1)\}$ of $\mathbb{R}^2$.

(c) Find the eigenvalues and eigenvectors of the matrix:

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

(d) (i) Find the product of all values of $(1+i\sqrt{3})^{\frac{3}{4}}$

(ii) If $x+\frac{1}{x}=2\cos\theta$ and θ is real, prove that $x^n+\frac{1}{x^n}=2\cos n\theta$. 3+2=5

(e) (i) Prove that the product of any m consecutive integers is divisible by m.

(ii) If a is prime to b, prove that a^2 is prime to b and a^2 is prime to b^2. 3+2=5

(f) Solve, if possible, the system of equations

$x_1+2x_2-x_3=10$

$-x_1+x_2+2x_3=2$

$2x_1+x_2-3x_3=8$

3. Answer *any one* question: 10×1=10

(a) (i) Find the remainder when $1!+2!+\cdots+50!$ is divided by 15.

(ii) If Z is a non-zero complex number and p, q, m and n are positive integers, where $\frac{p}{q}=\frac{m}{n}$ with $\gcd(m,n)=1$, then $Z^{\frac{p}{q}}=Z^{\frac{m}{n}}$.

(iii) Solve the equation using Cardan's method $x^3-18x-35=0$ 2+3+5=10

(b) (i) State Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{pmatrix}$ and hence compute A^{-1}.

(ii) The matrix of a linear mapping $T:\mathbb{R}^3\to\mathbb{R}^2$ relative to the ordered bases $\{(0,1,1),(1,0,1),(1,1,0)\}$ of $\mathbb{R}^3$ and $\{(1,0),(1,1)\}$ of $\mathbb{R}^2$ is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$.Find the linear transformation T . Find the matrix of T relative to the ordered bases $\{(1,1,0),(1,0,1)(0,1,1)\}$ of $\mathbb{R}^3$ and $\{(1,1),(0,1)\}$ of $\mathbb{R}^2$. (1+3+1)+(3+2)=10

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B.Sc. 3rd Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 32115 **Course Code : SHMTH-305/SEC-1**

Course Title: Programming Using C (New)

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) What are the differences between low level language and high level language?

(b) Express the following expression as valid *C*-expression: $\text{Sec}\, X + \frac{1}{\text{Sin}\, x + \text{Cosec}\, x}$.

(c) Give reasons why the following constants are not granted as *C* real constants:

(i) 3A2·8B (ii) 13E13 (iii) 1·3E1·3 (iv) –132

(d) What is the difference between "`printf`" and "`fprintf`" functions?

(e) Point out errors, if any in *C*:

(i) `3·14*r*r*h = Vol-of-cyl;`

(ii) `y-inst=rate of interest*amount in rs;`

(f) Find the output

```
int a=5, b=2; int c; c=a%b; printf ("%d ", c);
```

(g) How many times will the following loop execute?

```
int i; for(i=0; i<10; i++)
          {
          Printf("%d\n", i++);
          I=++i;
          }
```

(h) What is the difference between $++i$ and $i++$?

2. Answer *any four* questions: 5×4=20

(a) (i) What are the differences between the break and continue statements in C programming?

(ii) Write a do-while loop to evaluate and print the values of the quadratic polynomial $y = x^2 + 10x - 11$ for the values of $x = 0{\cdot}0$ to $1{\cdot}0$ with step 0·2. 2+3=5

(b) Write a program in *C* language to find the first 15 terms of Fibonacci sequence. 5

(c) Write a *C* program to find the GCD of two positive integers. 5

(d) Write a *C* program to find the sum $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$. 5

(e) Write a program to check whether a number is prime or not (using break statement). 5

(f) Write a *C* program to check leap year. 5

3. Answer *any one* question: 10×1=10

(a) (i) Write a complete *C* program to calculate the following function for different argument values:

$$\left.\begin{aligned} f(x) &= x^2+\text{Sin }2x && \text{when } x<3 \\ &= 10{\cdot}3 && \text{when } x=3 \\ &= x^3-\text{Cos }3x && \text{when } x>3 \end{aligned}\right\}.$$

(ii) What is the difference between local and global variable declaration? What is the purpose of using 'return' statement in *C* function subprogram? 6+2+2=10

(b) (i) Write a complete *C* program to display the maximum and minimum numbers from a series of numbers by using array variables.

(ii) What would be the output of the following program?

```
main()
{
  int i=4,j=-1,k=0,w,x,y,z;
  w=i||j||k;
  x=i&&j&&k;
  y=i||j&&k;
  z=i&&j||k;
  printf("\nw=%dx=%dy=%dz=%d", w,x,y,z);
}.
```

5+5=10

B.Sc. 3rd Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 32115 **Course Code : SHMTH-305/SEC-1**

Course Title: Logic and Sets

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Show that $R = \{(a, b): a \in \mathbb{Z},\ b \in \mathbb{Z},\ (a - b) \text{ is an even integer}\}$ is an equivalence relation on $\mathbb{Z}$.

(b) Show that $A = \{2,3,4,5\}$ is not a subset of $B = \{x: x \in \mathbb{N}, x \text{ is even}\}$.

(c) Find the elements of the set $A = \{\{1,2,3\}, \{4,5\}, \{5,7,8\}\}$.

(d) Determine the power set of $A = \{a, b, c, d\}$.

(e) Find the number of relations from $A = \{a, b, c\}$ to $B = \{1,2\}$.

(f) Prove that $(A \times B) \cap (A \times C) = A \times (B \cap C)$.

(g) Construct a truth table for the statement formula $\sim (\sim p \wedge q)$.

(h) Find the negation of the following quantified predicates: $(\exists x \in D)(x + 2 = 7)$.

2. Answer *any four* questions: 5×4=20

(a) It is known that in a university 60% of professors play tennis, 50% of them play bridge, 70% jog, 20% play tennis and bridge, 40% play bridge and jog and 30% play tennis and jog. If someone claimed that 20% professors jog and play tennis and bridge, would you believe his claim? Why? 5

(b) (i) Show that we can have $A \cap B = A \cap C$ without $B = C$

(ii) Prove that $(A \cup B)\backslash(A \cap B) = (A\backslash B) \cup (B\backslash A)$. 2+3=5

(c) Suppose $\mathbb{N} = \{1,2,3, \ldots.\}$ is the universal set and

$A = \{x: x \leq 6\}, B = \{x: 4 \leq x \leq 9\}, C = \{1, 3, 5, 7, 9\}, D = \{2, 3, 5, 7, 8\}$.

Then find $A \cap B, B \cup C, A \cap (B \cup D), (A \cap B) \cup (A \cap D)$. 1+1+1+2=5

(d) If R be an equivalence relation on the set A, then show that R^{-1} is also an equivalence relation on A. 5

(e) (i) Construct a truth table for the statement form:

$$(p \wedge q) \vee \sim r.$$

(ii) Find the negation of the following statement:

$\exists x p(x) \wedge \exists\, y\, q\, (y)$. 3+2=5

(f) Construct the table for

(i) $(a \vee b) \leftrightarrow [((\sim a) \wedge c) \longrightarrow (b \wedge c)]$

(ii) $p \vee q$. 3+2=5

3. Answer *any one* question: 10×1=10

(a) In a class of 80 students, 50 students know English, 55 know French and 46 know German language. 37 students know English and French, 28 students know French and German. 7 students know none of the language. Find out

(i) How many students know all the three languages?

(ii) How many students know exactly two languages?

(iii) How many students know only one language? 3+3+4=10

(b) (i) Let *p, q, r* be statements. Then show that $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$ holds by truth table.

(ii) A relation ρ is defined on the set $\mathbb{Z}$ by "$a\rho b$ if and only if $ab > 0$" for $a, b \in \mathbb{Z}$. Examine if ρ is reflexive, symmetric and transitive.

(iii) Let ρ be a relation on a set *A*. Then prove that ρ is symmetric if and only if $\rho^{-1} = \rho$.

4+3+3=10

B.Sc. 3rd Semester (Programme) Examination, 2019-20

MATHEMATICS

Course ID : 32118 **Course Code : SP/MTH-301/C-1C**

Course Title: Algebra

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual mainly.

1. Answer *any five* questions: 2×5=10

(a) Find the value of $\left(-1+\sqrt{3}i\right)^{12}$.

(b) Transform the equation to remove the squre term from the equation $x^3+9x^2+15x-25=0$

(c) State first and second principle of induction.

(d) Define rank of a matrix and hence find the rank of $\begin{pmatrix}1 & 2\\ 2 & 4\end{pmatrix}$.

(e) If *a, b, c* be positive real numbers, not all equal, prove that $(a+b)(b+c)(c+a)>8abc$.

(f) A relation ρ is defined on the set ℤ by $a\,\rho\,b$ if and only if $a-b$ is divisible by 5 for *a, b* ∈ ℤ. Show that ρ is an equivalence relation on ℤ.

(g) Let $f,g:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=4x-1$ and $g(x)=x^2+2$. Find *fog* and *gof*.

(h) Define $T:\mathbb{R}^3\to\mathbb{R}^2$ by $T(x,y,z)=(x-y,2z)$, show that *T* is linear.

2. Answer *any four* questions: 5×4=20

(a) (i) Solve the equation $16x^4-64x^3+56x^2+16x-15=0$. Whose roots are in arithmetic progression.

(ii) Using Descarte's rule of sign, find the number of complex root of the equation $2x^4+x^2+7x-6=0$. 3+2=5

(b) The matrix of a linear transformation $T:\mathbb{R}^3\to\mathbb{R}^3$ with respect to the ordered basis $\{(0,1,1),(1,0,1),(1,1,0)\}$ is given by $\begin{pmatrix}0 & 3 & 0\\ 2 & 3 & -2\\ 2 & -1 & 2\end{pmatrix}$. Find *T*.

(c) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix}1 & -1 & 2\\ 2 & -2 & 4\\ 3 & -3 & 6\end{pmatrix}$.

(d) (i) Let *a, b, c, d* be positive real numbers, not all equal. Use Cauchy–Schwarz inequality to show that $(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)>16$.

(ii) If *a, b, c* be positive real numbers and $abc=k^3$, prove that $(1+a)(1+b)(1+c)\geq(1+k)^3$. 2+3=5

(e) (i) Use first principle of induction to show that $7^{2n} + 16n - 1$ is divisible by 64, $\forall n \in \mathbb{N}$.

(ii) Use theory of congruence, find the remainder when 3^{36} is divided by 77. 3+2=5

(f) Solve the system of equations

$$x + 2y + z = 1$$
$$3x + y + 2z = 3$$
$$x + 7y + 2z = 1$$

3. Answer *any one* question: 10×1=10

(a) (i) Find the remainder when 3^{36} is divided by 77.

(ii) Solve the equation using Cardan's method $x^3 - 27x - 54 = 0$.

(iii) Solve the equation $x^8 + x^7 + x^6 + \cdots + x + 1 = 0$ 2+5+3=10

(b) (i) State Cayley-Hamilton theorem.

(ii) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. Hence find A^{-1}.

(iii) Obtain a row-echelon matrix which is equivalent to $\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$.

1+(3+2)+4=10

SP-III/Math-304/SEC-1/19

B.Sc. 3rd Semester (Programme) Examination, 2019-20

MATHEMATICS

Course ID : 32110 **Course Code : SPMTH/304/SEC-1**

Course Title: Logic and Sets

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Construct a truth table for the statement formula: $(p \to q) \to (\sim q \to \sim p)$.

(b) If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4, 5\}$, then find $A \Delta B$ where Δ is the symmetric difference between two sets.

(c) A relation ρ is defined on the set $\mathbb{Z}$ by "$a \, \rho \, b$ if and only if $ab > 0$" for $a, b, \in \mathbb{Z}$. Examine if ρ is reflexive.

(d) Find $\bigcap_{n \in \mathbb{N}} I_n$ where $I_n = \left(0, \frac{1}{n}\right), n \in \mathbb{N}$.

(e) If $A = \{1, 2\}, B = \{1, 2, 3\}$, find $(A \times B) \cap (B \times A)$.

(f) Using Venn-diagram prove that $(A - C) \cup (B - C) = (A \cup B) - C$.

(g) Write down the truth table for biconditional proposition.

(h) What is the negation of the proposition "Some people have no scooter".

2. Answer *any four* questions: 5×4=20

(a) (i) Construct a truth table for the statement form: $(p \wedge q) \vee \sim r$

(ii) Give the negation of the following statements:

$p : 2 + 3 > 1$

q : It is cold. 3+2=5

(b) (i) If p and q are propositions, then show that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$.

(ii) Write negation of the following statement. If I am ill, then I cannot go to University.

3+2=5

(c) (i) The relation ρ defined on the set ℤ by "$a\rho b$ if and only if $a|b$". Examine if ρ is partial order relation.

(ii) Find the equivalence classes determined by the equivalence relation ρ on ℤ defined by "$a\rho b$ if and only if $a-b$ is divisible by 5" for $a, b \in$ ℤ. 3+2=5

(d) A, B, C are subsets of a universal set S.

(i) Prove that $(A \cup B) \cap (A \cup B') \cap (A' \cup B) = A \cap B$.

(ii) If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, prove that $B = C$. 3+2=5

(e) A, B, C are subsets of a universal set S. Prove that

(i) $[A \cap (B \cup C)] \cap [A' \cup (B' \cap C')] = \phi$

(ii) $(A - B) \times C = (A \times C) - (B \times C)$ 2+3=5

(f) Each student of a class speaks at least one language of Hindi and Bengali. If 40 speak Bengali, 16 speak Hindi and 8 speak both Hindi and Bengali; find the number of students in the class.

3. Answer *any one* question: 10×1=10

(a) (i) Let ρ be an equivalence relation on a set S and $a, b \in S$. Then $cl\,(a) = cl\,(b)$ if and only if $a\ \rho\ b$.

(ii) Prove that $A \cap (B \Delta C) = (A \cap C)$ when $A.B.C.$ are subsets of a universal set ∪. 5+5=10

(b) (i) Show that the propositions $\sim(p \wedge q)$ and $\sim p \vee \sim q$ are logically equivalent.

(ii) Construct the table for $(a \vee b) \leftrightarrow \big[\big((\sim a) \wedge c\big) \rightarrow (b \wedge c)\big]$.

(iii) Let R and S be the following relations on

$A = \{1, 2, 3\}$

$R = \{(1,1), (1,2), (2,3), (3,1), (3,3)\}$

$S = \{(1,2), (1,3), (2,1), (3,3)\}$

Find $R \circ S$, R^c 3+4+(2+1)=10

SH-III/Physics/503/DSE-1/19

B.Sc. 3rd Semester (Honours) Examination, 2019-20

PHYSICS

Course ID : 32411 **Course Code : SH/PHS/301/C-5**

Course Title : Mathematical Physics-II

Time: 1 Hours 15 Minutes **Full Marks: 25**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Section–I

Answer *any five* questions. 1×5=5

1. (a) State Cauchy's Residue theorem.

(b) Evaluate $\int_{-\infty}^{\infty} x\,\delta(x-4)dx$.

(c) Find the nature of the singularity of the function

$f(z) = e^{\frac{1}{z-2}}$ at z = 2.

(d) What do you mean by unitary matrix?

(e) Find the probability of drawing 2 aces in succession from a pack of 52 cards.

(f) Show that if a given co-ordinate is cyclic in the Lagrangian, it will also be cyclic in Hamiltonian.

(g) What are the properties of eigenvector and eigenvalues of Harmitian matrix?

(h) What do you mean by a pole?

Section–II

Answer *any two* questions. 5×2=10

2. (a) Prove that, if $\hat{A}$ is a linear operator and is invertible then $\hat{A}^{-1}$ is also a linear operator.

(b) Define the norm of a vector in linear vector space. What are their properties? 2+3=5

3. (a) Show that every diagonal element of a skew-Harmitian matrix is either zero or a pure imaginary number.

(b) Given $A = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$

Show that $U = [I-A][I+A]^{-1}$ is unitary. 2+3=5

4. If the probability of a bad reaction from a medicine is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction. 5

5. Derive canonical equation of motion from variational principle. 5

Section–III

Answer *any one* question. 10×1=10

6. (a) Find the square root of *i*.

(b) Using residue theorem evaluate $I = \int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ 4+6=10

7. (a) Solve the following equations by matrix method

$$x - 2y + 3z = 5$$
$$4x + 3y + 4z = 7$$
$$x + y - z = -4$$

(b) Are the following vectors linearly dependent or not?

$$x_1 = (3, 2, 7)$$
$$x_2 = (2, 4, 1)$$
$$x_3 = (1, -2, 6)$$

(c) Show that

(i) $\delta[c(x-a)] = \frac{1}{|c|}\delta(x-a)$

(ii) $\delta[(x^2 - a^2)] = \frac{1}{2a}[\delta(x-a) + \delta(x+a)], a > 0$ 5+2+3=10

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B.Sc. 3rd Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 32111 **Course Code : SHMTH-301-C-5**

Course Title: Theory of Real Functions and Introduction to Metric Spaces

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Show that $\operatorname*{Lt}_{x\to 0}\frac{1}{x}\sin\frac{1}{x}$ does not exist.

(b) Prove or disprove: The intermediate value theorem is applicable to the function $f(x)=\begin{cases}2x+1, & x\in(0,1]\\ 0, & x=0\end{cases}$.

(c) Show that on the real numbers with the usual metric, the set of natural numbers is closed.

(d) Test if Lagrange's mean value theorem holds for the function $f(x)=|x|$ in the interval $[-1,1]$.

(e) Prove or disprove:

If $f(x)$ and $g(x)$ be two functions such that $\lim_{x\to a}[f(x)+g(x)]$ exists, then $\lim_{x\to a}f(x)$ and $\lim_{x\to a}g(x)$ exist.

(f) Show that the function $f(x)=\cos\frac{1}{x}, 0<x<1$, is not uniformly continuous on $(0,1)$.

(g) Find the diameter of the set

$\left\{(x,y): 0<x<\frac{\pi}{2},\ y=\cos x\right\}$ with respect to usual metric on R^2.

(h) Show that in a discrete metric space (X,d), every subset of X is open set.

2. Answer *any four* questions: 5×4=20

(a) (i) Prove that $\operatorname*{Lt}_{x\to a}f(x)$ exists and is equal to l if and only if $\operatorname*{Lt}_{x\to a^+}f(x)$ and $\operatorname*{Lt}_{x\to a^-}f(x)$ both exist and are equal to l.

(ii) If a function f is derivable in a closed interval $[a,b]$ and $f'(a)\neq f'(b)$, and k is a real number lying between $f'(a)$ and $f'(b)$, then show that there exists at least one point $c\in(a,b)$ such that $f'(c)=k$. 3+2=5

(b) If f' exists and is bounded on some interval I, then prove that f is uniformly continuous on I.

(c) Let (X, d) be a metric space and ρ be a function on X x X defined by $\rho(x, y) = \min\{1, d(x, y)\}$ for all $x, y \in x$. Show that ρ is a metric on x.

(d) (i) Let $f: [a, b] \to R$ be a function and $c \in [a, b]$ and for every sequence $\{x_n\}$ in $[a, b]$ which converges to 'c', we have $\lim_{n \to \alpha} f(x_n) = f(c)$, then show that $f(x)$ is continuous at $x = c$.

(ii) Write $\in -\delta$ definition of a function not to be uniformly continuous. 3+2=5

(e) (i) A function $f: R \to R$ is continuous on R. Prove that the set $S = \{x \in R: f(x) > 0\}$ is an open set in R, where R is the set of Reals.

(ii) Prove or disprove:

Every continuous function is always monotonic. 3+2=5

(f) (i) Show that a subset A of a metric space (X, d) is closed if and only if $X - A$ is open set.

(ii) In mean value theorem,

$f(h) = f(0) + h\, f'(\theta h), 0 < \theta < 1, \lim_{h \to 0} \theta = \frac{1}{2}$, when $f(x) = \cos x$.

3. Answer *any one* question: 10×1=10

(a) (i) Let $f(x) = \begin{cases} x^p \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ Obtain p such that $f(x)$ is continuous at $x = 0$ and $f(x)$ is differentiable at $x = 0$.

(ii) Let f be continuous on $[a - h, a + h]$ and derivable on $(a - h,\ a + h)$. Prove that there exists a real number θ $(0 < \theta < 1)$ for which

$f(a + h) - 2\, f(a) + f(a - h) = h\, [f'(a + \theta h) - f'(a - \theta h)]$.

(iii) Give an example with justification to show that an open set may not be an open sphere.

5+3+2=10

(b) (i) When is a function $f(x)$ said to have local maxima at $x = a$? Does $f'(a) = 0$ always imply existence of an extremum of f at $x = c$? Justify.

(ii) Expand $\sin\theta$ as a finite series of expansion in 'θ'.

(iii) When a function f is said to be convex function on an interval $[a, b]$? If f is convex in $[a, b]$, then show that f'' is non-negative in $[a, b]$.

(iv) Let (X, d) be a metric space and let $x, y (\neq x) \in X$. Prove that there is a nbd. M of x and a nbd N of y such that $M \cap N = \varphi$. 2+2+4+2=10

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